SPECTRAL METHODS IN OFFSHORE DESIGN

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The continuing drive to cut costs and produce more efficient designs, has accentuated the need for a more accurate representation of dynamic effects and a fuller, more comprehensive, representation of environmental loads. It is particularly important for dynamically active fixed structures in deep water where fatigue performance has become a governing factor in the design of some major nodes.

Floating structures are also becoming economically attractive with the introduction of flexible riser technology. This area of design is one where probabilistic or spectral methods have been traditionally used, as rigid body resonances are often within frequency ranges of appreciable wave energy.

It must be borne in mind when faced with the (apparent) complexities of the spectral approach, that conventional deterministic design methods have their basis in the probabilistic approach. How can "design" loads be determined without consideration of their probability of occurrence and their variability?

This paper is an attempt to describe the spectral method and give an overview of its use in the design of offshore structures.

1.0 Summary of the Spectral Method * (figure 1)

Consider the random variable "x" (e.g. water surface elevation) which is the input to an offshore jacket analysis. Assume that the value of an output variable "y" is required (say the displacement of the topsides centre of gravity).
The time history x(t) may be broken down into a number of harmonic components by Fourier decomposition.
Assume each component has frequency ω_n and amplitude $X(\omega_n)$ as shown. The original section of x(t) may be re-constructed by the addition of these components.
If we now assume that the system being excited is a "linear" system, then each harmonic component of x(t) gives rise to a component of y(t) with the same frequency.

The amplitude of this component of y may be obtained by multiplying the corresponding component of x by the ordinate of the "Transfer Function" $T(\omega_n)$ corresponding to the frequency ω_n

i.e. $Y(\omega_n) = T(\omega_n)\,X(\omega_n)$

n.b. T may be complex giving a phase shift.

* Apologies to Mathematicians and other purists.

The time history of the output $y(t)$ may now be synthesised from its components $Y(\omega_n)$.

Another representation of the relative magnitudes of the components $X(\omega_n)$ of $x(t)$, is the power spectral density (or spectrum) of x and is shown as $Sxx(\omega)$. This function of frequency indicates the distribution of energy by frequency in $x(t)$. For our example of water surface elevation, $Sxx(\omega)$ is the familiar wave energy spectrum.

It can be shown that:- $Syy(\omega) = |T(\omega)|^2 Sxx(\omega)$

Furthermore the mean square (r.m.s.)^2 value of x or variance σ_x^2 of x is given by the area under the spectral curve.

$$\sigma_x^2 = \int_0^\infty S_{xx}(\omega)\,d\omega \qquad \text{(one sided spectrum)}$$

When this r.m.s. value of x is known, it is then possible to attach probabilities to certain values of x being observed if a probability density function is known, derived or assumed.

2.0 The System to be Analised.

Figure 2 shows schematically, the stages of analysis of an offshore structure. The left hand column refers to the deterministic approach and the right hand column represents the corresponding spectral approach.

Note that the deterministic approach starts with a probabilistic derivation of "design" waves representing either extreme storm or representative fatigue waves. The wave amplitude and period give rise to expressions for water particle velocities v and accelerations $\dot{v}$ of the form $v = Va$ the corresponding spectral formula $Svv(\omega) = |V(\omega)|^2 Saa(\omega)$ gives the spectrum of v in terms of the corresponding sea-state "wave spectrum" $Saa(\omega)$ and $V(\omega)$ now treated as a transfer function. The transfer functions relating v's to member forces f, f's to nodal displacements x and x's to hot-spot stresses s can be carried over to the spectral column to give relations between the corresponding power spectral densities or spectra. Ending with a spectrum of stresses at each hot-spot, which indicates the magnitude of stress range (or amplitude) for each input frequency.

Integration of $Sss(\omega)$ gives the variance or mean square value σ_s^2 for the sea state considered.

The probability density function p.d.f. then enables stress ranges to be evaluated with their associated probability of occurrence. This p.d.f. is related to the p.d.f. of the input variable (here surface elevation) and its peaks. Normally this p.d.f. is assumed to be the Rayleigh distribution:-

$$p(x) = \frac{x}{\sigma_{x}^2} \exp\left(\frac{-x^2}{\sigma_{x}^2}\right)$$

being the distribution of peaks of the normally distributed (Gaussian) water surface elevation.

The problem is now reduced to that of building the transfer functions linking the variables a, f, x and s.

(This treatment is necessarily simplified due to the multi-degree of freedom nature of jacket stiffness analysis.)

3.0 Case Studies

This section is a brief overview of the use of spectral techniques in offshore design.

3.1 Treatment of Non-linearities

For fixed structures, wave forces are calculated using the Morison equation, which contains non-linear terms associated with drag loading and finite wave height (wave climbing effects). In calculating the transfer function F relating v and f, a number of waves of different frequency . are used. The transfer function is hence calculated point by point and strictly speaking represents the member forces for waves of unit amplitude.
If a wave height is associated with each frequency and resulting force divided by this height, then a modified transfer function may be built up which allows for wave climbing and drag loading effects. Not true

Non-linear pile/soil effects may be accounted for in the transfer function between f and x by choosing a representative pile stiffness which corresponds to the frequency considered.

3.2 Fatigue

A spectral fatigue analysis (static or dynamic) is usually performed when the sway natural period of a fixed jacket is above 3 seconds. This is because representative waves in a deterministic analysis are often dangerously close in frequency to structural resonance. For a lightly damped structure (2% of critical), peak dynamic amplifications of up to 25 times can be obtained giving a damage contribution of 15,625 times the static contribution (T S/N curve). Clearly the choice of representative waves in this range is unacceptable and hence a spectral approach is necessary to smooth out the peaks and give a representation across the full frequency range.

In fact these short, low period waves will be short-crested, hence the full width of the jacket may not be loaded at the same time giving reduced response at resonance (incoherent loading). This short-crestedness or "wave spreading" may be represented using a directional spectrum. This may be seen from consideration of two over-lapping trains of long-crested waves travelling in slightly different directions.

3.3 Earthquake Analysis

Earthquake analysis is another obvious candidate for the spectral approach, as peak forcing occurs in the range 0.1 to 2 seconds where local structural resonance will occur. New jackets in the Norwegian sector are now required to be evaluated against various earthquake criteria. In addition reliable time histories of ground motion are not available in the U.K. and hence a spectral approach is appropriate using modal superposition. The "loading" is here the inertia and added inertia of the structure resisting ground accelerations. For the purposes of analysis the process is assumed to be stationary i.e. with constant statistical properties.

3.4 Gust Loading - flare boom analysis

The wind load on a typical flare boom is in two components, one due to the steady wind component V and another due to the fluctuating, gust or turbulent component v. Even though the loading is drag dominated, the in-line wind gust force is nearly linear in v, as v<<V. Furthermore the higher frequency components of turbulence have shorter characteristic length and hence these components are incoherent across the height of the flare boom. To take this into account, a filter function, the aerodynamic admittance .... is used to modify the gust spectrum to reduce the nett contribution of higher frequency loading. Flare booms are also very lightly damped (<1% of critical) which makes a spectral approach necessary.

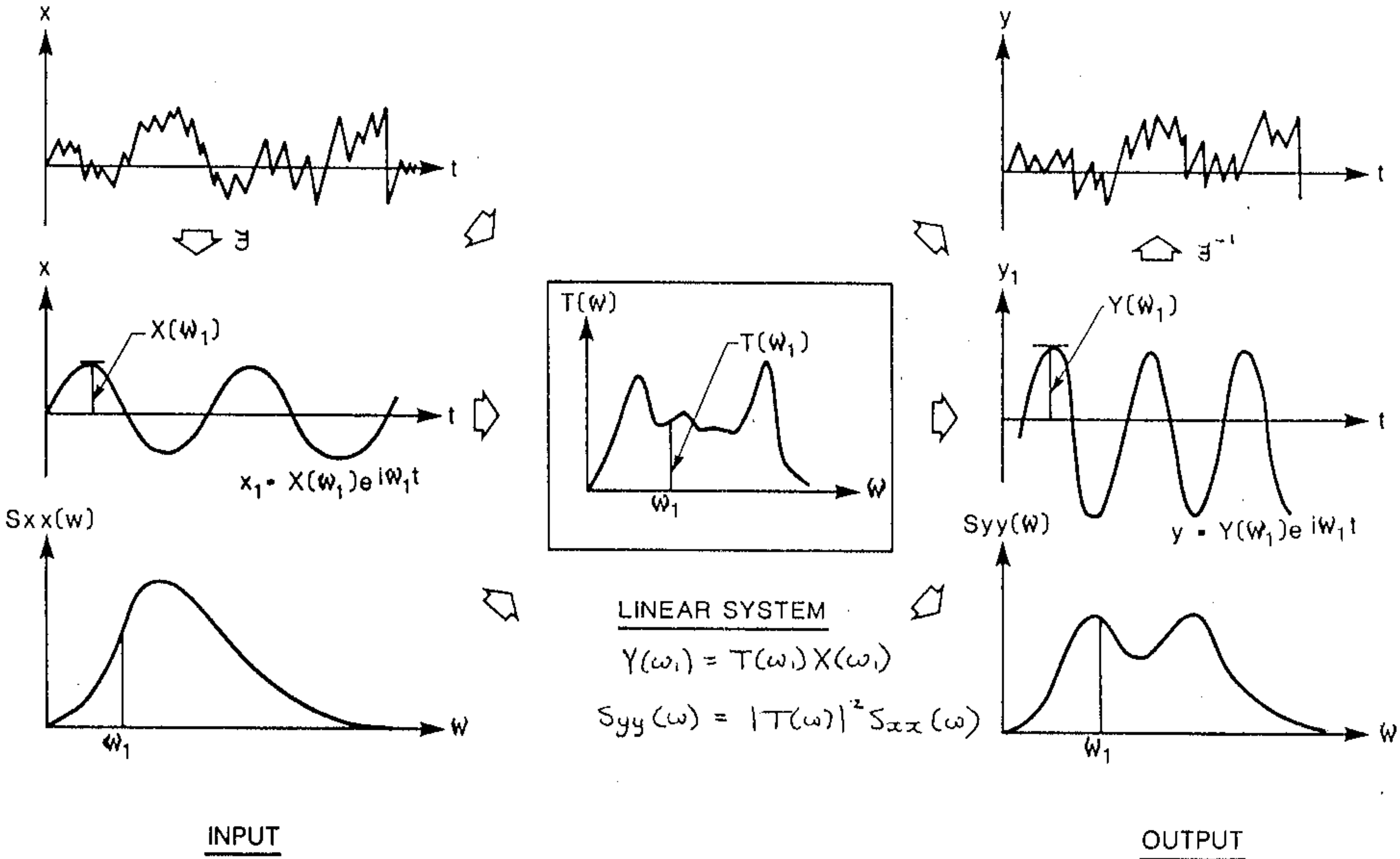
3.5 Floating Structures.

Semi-submersibles have traditionally been examined using the spectral approach, because of the rigid body resonances, particularly heave, which occur in the wave frequency range. In order to represent the true motion response characteristics, transfer functions, (or response amplitude operators r.a.o's) are represented by irregular sea r.a.o's. This has led to a routine use of spectral methods in fatigue and down-time evaluation.

With the increased use of flexible risers, the probabilistic approach is often used to evaluate peak stresses and relative displacements in order to detect riser clashes. In general where it is necessary to consider the combined effect of a number of random variables, it is conventional to derive the statistical measures of each variable and extrapolate some combination of these to give extreme combined values. This is more satisfactory than extrapolating each variable and combining the extremes. This approach also has its use in multiple body problems such as semisubmersible or tanker offloading systems.

4.0 Summary of the use of Spectral Techniques

ANALYSIS	Reason for use of spectral Technique
Fatigue	Dynamic Effects Full Environmental Representation
Earthquake Analysis	Dynamic Effects No reliable time histories
Wind Loading - Flares	Incoherent Loading, Dynamics
Wave Spreading	Incoherent Loading
Floating and articulated Structures	Dynamic effects
Jacket Transportation	Interacting Random Variables
Riser Analysis	Interacting Random Variables



SPECTRAL ANALYSIS SUMMARY OF METHOD

Figure 1

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