On the Hydrostatics of Floating Bodies with Articulated Appendages

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SUMMARY: The conventional theory of rigid body hydrostatic stability is extended to floating bodies which include freely articulated members as part of the buoyancy and stability contributing structure. The calculations are illustrated using an advanced semisubmersible design which utilises articulated members to achieve a very high deck payload. It is shown that the presence of the articulated members does not degrade the hydrostatic stability to a significant extent and actually enhances it for low angles of vessel inclination. The impact of the articulated members on classification society certification requirements is discussed.

1. INTRODUCTION

The hydrostatic stability of marine vehicles floating at the air water interface has been fully investigated and extensively documented in naval architecture literature. Ramsey(1) presented an early unified treatment of the theory. These theoretical investigations and results have been concerned with the hydrostatic stability of marine vehicles which can be regarded as rigid bodies in the context of the hydrostatics calculations. The upsurge of offshore oil exploitation and production activities in the Gulf of Mexico and currently in the North Sea over the last two decades has required a variety of novel floating marine vehicles such as semisubmersibles to be developed and used as offshore work platforms. These vehicles can still be considered as rigid bodies but the detailed application of hydrostatics principles to these vessels has had to be modified, particularly for large angles of heel. However, a further class of semisubmersible offshore work platforms is being developed to support floating production and drilling activities for deep water and in remote North Atlantic, Northern North Sea and Arctic locations. This class of marine vehicles employs articulations in the structure to achieve the high metacentric heights required for a large deck payload without the penalty of structure weight that would be incurred in the absence of the articulations. Fig. 1 shows the general arrangement of such a vessel where the buoyancy chambers B support the deck D through a space frame structure S with the primary contribution to the hydrostatic stability being supplied by the six articulated stabilisers, A, on the periphery of the vessel.

The presence of these articulations requires that the conventionally accepted theory of hydrostatic stability be extended to account for and calculate the effects of articulations in the structure. This paper describes such an extension both for small and large angles of inclination. The theory of rigid body hydrostatics is presented in brief detail so as to compare and contrast the differences with the hydrostatics results arising from the articulated body. The paper further highlights special considerations that must apply to a vessel with articulated appendages for calculating forces on the articulation joints and for intact and damaged stability.

2. FIRST ORDER THEORY FOR SMALL ANGLES OF INCLINATION

2.1 Conventional Rigid Body Hydrostatics

The rigid body hydrostatic stability analysis of an arbitrary body is briefly repeated here in the same manner as presented by Ramsey(1) in order to compare the rigid and articulated body analyses. Consider the arbitrarily shaped floating body shown in Fig. 2 with, for the moment, a rigid connection at the articulation.

Under static conditions, the body's immersed volume V and the consequential buoyancy force F_B will remain constant with the force F_B acting through the body's immersed centre of volume (or centre of buoyancy), B. A consideration of the hydrostatic stability of the body requires that inclination of the body about the Ox and Oy axes in the still water plane be considered. If the body is given a small rotation, β, about the

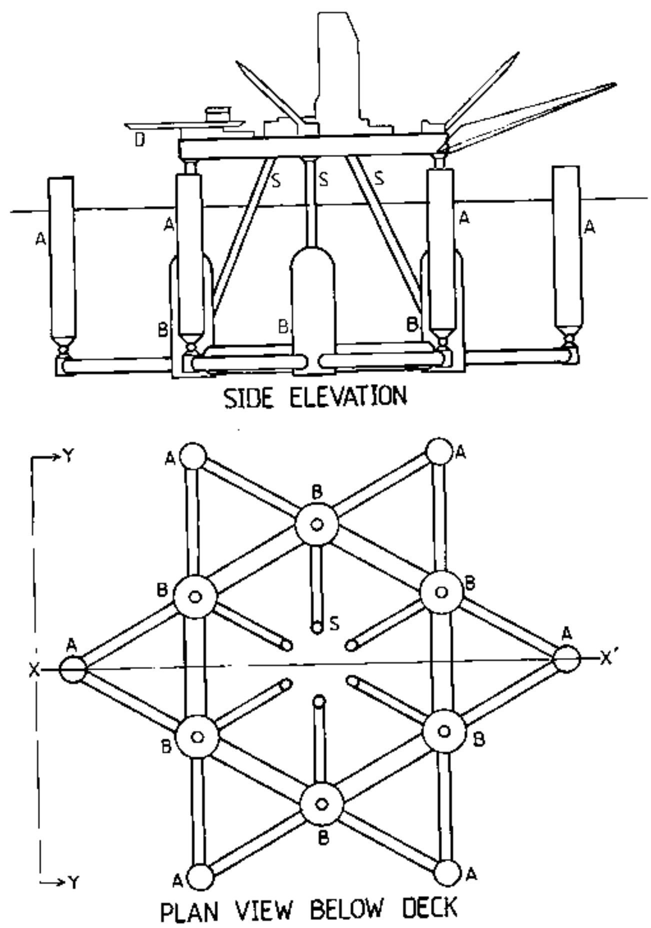

Fig. 1. Typical Vessel with Articulated Appendages

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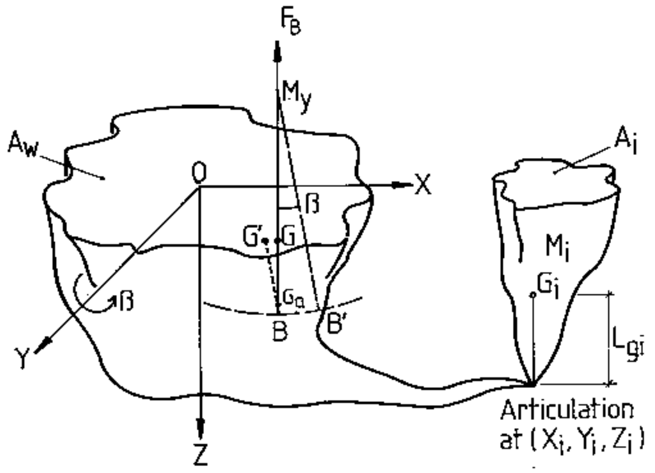

Fig. 2. An Arbitrary Rigid Body with an Articulated Appendage

Oy axis, the immersed volumes before and after the rotation must be equal if the buoyancy force which reacts the body weight is to remain constant. Equating the immersed volumes before and after rotation gives

$$\int_{A_w} z dA_W = \int_{A_w} (z - x\beta) dA_W \tag{1}$$

where $z = z(x, y)$ denotes the shape of the submerged body boundary and the integral on the right hand side includes the volume changes due to emergence and submergence of wedges of fluid displaced in the vicinity of the cut water plane area, A_W.

Then, for any non-zero small rotation about Oy,

$$\int_{A_w} x dA_W = 0 \tag{2}$$

and similarly for rotation about the Ox axis,

$$\int_{A_w} y dA_W = 0 \tag{3}$$

leading to the conclusion that the body can only rotate at constant displacement about a point which must be the centroid of the water plane area, the centre of flotation. However, it is the motion of the centre of buoyancy, B, as β varies which governs the stability of the floating body. If the coordinates of B are $(\bar{x}, \bar{y}, \bar{z})$ in the equilibrium static position ($\beta = 0$) and shift to point B' $(\bar{x}', \bar{y}', \bar{z}')$ after a small rotation β about the y axis, then the difference in the coordinates of the centre of buoyancy can be readily expressed as

$$\bar{x} - \bar{x}' = \frac{\beta}{V} \int_{A_w} x^2 dA_W \tag{4}$$

$$\bar{y} - \bar{y}' = \frac{\beta}{V} \int_{A_w} xy dA_W \tag{5}$$

and

$$\bar{z} - \bar{z}' = \frac{\beta^2}{2V} \int_{A_w} x^2 dA_W \tag{6}$$

Equation (6) leads to the well known result that the vertical movement of the centre of buoyancy is of second order and positive for small β so that the centre of buoyancy will, in general, move on a surface (the surface of buoyancy) which is horizontal at B and concave upwards. Since $\bar{y} = \bar{y}'$ for small β, equation (5) leads to the assertion that

$$\int_{A_w} xy dA_W = 0 \tag{7}$$

and that Oxy must be principal axes for the waterplane area. By consideration of the horizontal movement of the centre of buoyancy as given by equation (4), a metacentre M_y can be defined such that

$$BM_y = \frac{1}{V} \int_{A_w} x^2 dA_W \tag{8}$$

for rotation about axis Oy with

$$BM_x = \frac{1}{V} \int_{A_w} y^2 dA_W \tag{9}$$

for rotation about axis Ox.

For conventional rigid body hydrostatics in the absence of fluid tanks with a free surface or pendulum effects due to hanging weights, the centre of gravity position will remain fixed in the body. The metacentric heights GM_x and GM_y will then be a measure of the righting moments that will be exerted due to body rotations about the Ox and Oy axes respectively.

2.2 Rigid Body With Articulated Appendages

Now consider an arbitrary rigid body from which are mounted a number of articulated appendages each of which cuts through the waterplane area and has sufficient excess buoyancy (above self weight) to remain in a vertical equilibrium position. Fig. 2 shows a sketch of the body and just one of the articulated appendages mounted at position (x_i, y_i, z_i) with a waterplane area of A_i. The articulation is assumed to be universal such that the appendage is free to move in all vertical planes through (x_i, y_i, z_i). When the floating body is at rest, the net forces due to buoyancy and weight (with positive excess buoyancy) exerted by the articulated members are transmitted through the joint into the main structure with no modification. This implies that centre of buoyancy position is not influenced by whether or not the articulations are considered rigid although the orientation of the articulated member relative to the body will influence both the centre of buoyancy and centre of gravity positions.

As a result of a small rotation β about the Oy axis, the rigid body will have emerging and submerging wedges contributing to the hydrostatic forces whereas the articulated members will submerge or emerge vertically as cross-sections with local waterplane area, A_i. There will be an additional shift in centre of buoyancy due to the change of position of the submerged volumes in the rotated articulations. Furthermore, this rotation will induce asymmetric articulation waterplane areas about the Oy axis; thus causing a shift in centre of flotation which will induce a deficit in submerged volume and a consequent parallel sinkage with increasing angle of inclination, β.

The deficit in submerged volume is given by taking the difference of submerged volumes before and after a small rotation β about the Oy axis. This gives

$$\int_{A_w} z dA_W + \sum_i \int_{A_w} z_{ai} dA_W = \int_{A_w} (z - x\beta) dA_W + \sum_i \int_{A_w} [z_{ai} - (x_i + l_i\beta)\beta] dA_W \tag{10}$$

where $z_{ai} = z(x, y)$ describes the submerged shape of articulated member i and l_i is the draught of the member pivot. For symmetric dispositions of rigid and articulated members about the Oy axis

$$\int_{A_w} x dA_W = 0 \tag{11}$$

and

$$\sum_i \int_{A_w} x_i dA_W = 0 \tag{12}$$

Then from equation (10), the deficit of volume is

$$\beta^2 \sum_i l_i A_i \tag{13}$$

where A_i is the waterplane area of articulation number i. The parallel sinkage, p, is

$$p = \frac{\sum_i l_i A_i}{A_w + \sum_i A_i} \beta^2 \quad (14)$$

Thus the presence of symmetric articulations does shift the centre of flotation and induces a second order parallel sinkage.

Turning now to the position of the centre of buoyancy, the coordinates of the centre of buoyancy before inclination can be given by equations (15), (16) and (17) below with the effects of the articulations included.

Thus

$$\bar{x} = \frac{1}{V}\left[\int_{A_w} xz dA_w + \sum_i \int_{A_i} x z_{ai} dA_i\right] \quad (15)$$

$$\bar{y} = \frac{1}{V}\left[\int_{A_w} yz dA_w + \sum_i \int_{A_i} y z_{ai} dA_i\right] \quad (16)$$

$$\bar{z} = \frac{1}{V}\left[\int_{A_w} \tfrac{1}{2} z^2 dA_w + \sum_i \int_{A_w} \tfrac{1}{2} z_{ai}^2 dA_i\right] \quad (17)$$

After the small rotation, β, the centre of buoyancy, B, will be at coordinates given by

$$\bar{x}' = \frac{1}{V}\left[\int_{A_w} x(z - x\beta) dA_w + \sum_i \int_{A_i} (x_i + l_i\beta)(p - x_i\beta) dA_i + \sum_i \int_{A_i} (x_i + l_{bi}\beta) z_{ai} dA_i\right] \quad (18)$$

$$\bar{y}' = \frac{1}{V}\left[\int_{A_w} y(z - x\beta) dA_w + \sum_i \int_{A_i} y(p - x_i\beta) dA_i + \sum_i \int_{A_i} y z_{ai} dA_i\right] \quad (19)$$

and

$$\bar{z}' = \frac{1}{V}\left[\int_{A_w} \tfrac{1}{2}(z + x\beta)(z - x\beta) dA_w + \sum_i \int_{A_i} \tfrac{1}{2}\{z_{ai} + p + (x_i + l_i\beta)\beta\}\{z_{ai} + p - (x_i + l_i\beta)\beta\} dA_i\right] \quad (20)$$

where l_{bi} is the distance of the articulation centre of buoyancy above the pivot.

Then, from equations (17) and (20), the vertical movement of the centre of buoyancy is given by

$$\bar{z} - \bar{z}' = \frac{\beta^2}{2V}\left[\int_{A_w} x^2 dA_w + \sum_i \int_{A_i} x_i^2 dA_i\right] \quad (21)$$

by neglecting cubic and higher powers of β. This equation simplifies to

$$\bar{z} - \bar{z}' = \frac{\beta^2}{2V}\left[\int_{A_w} x^2 dA_w + \sum_i x_i^2 A_i\right] \quad (22)$$

which is also of second order for small β and is positive. Thus the surface of buoyancy for a rigid body with articulations is similar to that of the rigid body, that is, horizontal at B and concave upwards.

In the same way as before, for small β, B and B′ must lie in the same plane perpendicular to Oy. Thus

$$\bar{y} = \bar{y}' \quad (23)$$

and using equations (16) and (19),

$$\beta \int_{A_w} xy dA_w + \beta \sum_i x_i \int_{A_i} y dA_i - p \sum_i \int_{A_i} y dA_i = 0 \quad (24)$$

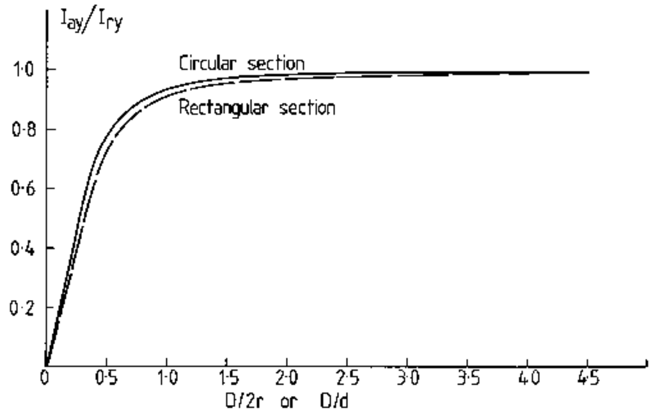

Fig. 3. Ratios of Articulated to Rigid Waterplane Areas

Again, if the articulated member waterplane areas are symmetric about the Ox axis, then

$$\sum_i x_i \int_{A_i} y dA_i = 0 \quad (25)$$

so that

$$\int_{A_w} xy dA_w = 0 \quad (26)$$

and Ox, Oy still remain principal axes of the total vessel waterplane area.

Now using Fig. 3 and equations (15) and (18), we get

$$BM_y = \frac{BB'}{\beta} = \frac{\bar{x} - \bar{x}'}{\beta} = \frac{1}{\beta V}\left\{\int_{A_w} \beta x^2 dA_w - \sum_i p x_i A_i - \sum_i \beta p l_i A_i + \sum_i \beta x_i^2 A_i + \sum_i \beta^2 x_i l_i A_i - \sum_i \beta l_{bi} V_i\right\} \quad (27)$$

where V_i is the initial submerged volume of the ith articulation. For initial symmetry of waterplane areas

$$\sum_i p x_i A_i = 0$$

and

$$\sum_i \beta^2 x_i l_i A_i = 0$$

$$\therefore BM_y = \frac{1}{V}\left\{\int_{A_w} x^2 dA_w + \sum_i x_i \int_{A_i} x dA_i - \sum_i l_{bi} V_i\right\} \quad (28)$$

if β^2 and higher powers are ignored.

If $I_{yy} = \int_{A_w} x^2 dA_w$, for the rigid part of the body,

and

$$I_{ay} = \sum_i x_i \int_{A_i} x dA_i \quad (29)$$

is due to the articulations, then

$$BM_y = \frac{I_{yy} + I_{ay}}{V} - \frac{\sum_i l_{bi} V_i}{V} \quad (30)$$

Similarly,

$$BM_x = \frac{I_{xx} + I_{ax}}{V} - \frac{\sum_i l_{bi} V_i}{V} \quad (31)$$

where

$$I_{ax} = \sum_i y_i \int_{A_i} y dA_i. \quad (32)$$

Unlike the conventional rigid body, however, the articulated vehicle also has an inherent horizontal shift in the centre of gravity position due to rotation of the articulations. This shift contributes to additional stability by increasing the effective value of GM as illustrated in Fig.2. The horizontal shift in centre of gravity position GG′ can be obtained by taking moments of forces due to mass about an axis through G perpendicular to the Oxz plane. This gives

$$MgGG' = g\Sigma m_i l_{gi}\beta + g\Sigma m_i s_i \tag{33}$$

where M is the total vessel mass, m_i are the masses of each articulated member with its local centre of gravity position G_i at a distance of l_{gi} above the articulation point (x_i, y_i, z_i) and a horizontal distance of s_i from the vertical through G. The second term on the right-hand side of equation (33) is zero for articulations that are symmetrically disposed about the vessel planes of symmetry. Thus the increment in metacentric height due to centre of gravity shift can be written for small β as

$$GG_a = \frac{GG'}{\beta} = \frac{\sum_i m_i l_{gi}}{M} \tag{34}$$

and the effective metacentric height can be written as

$$G_aM_y = BM_y - BG + GG_a$$
$$= \frac{I_{yy} + I_{ay}}{V} + \frac{\sum_i m_i l_{gi}}{\rho V} - \frac{\sum_i l_{bi} V_i}{V} - BG \tag{35}$$

where BG is the distance between the centres of buoyancy and gravity at zero angle of inclination.

2.3 Numerical Results

It is of interest to quantify the effects of articulated appendages by comparing the relative influence on distance BM of rigid and articulated members of equivalent waterplane area.

For a rectangular waterplane section of dimensions b and d, with its centroid a distance D (parallel to dimension d) away from the vessel axis, the rigid body second moment of waterplane area is

$$I_{ry} = \int_{A_w} x^2 dA_w = \frac{bd^3}{12} + bd.D^2 \tag{36}$$

whereas the articulated member value is

$$I_{ay} = x_i . \int_{A_i}^{L} x dA_i$$
$$= D\int_{D-d/2}^{D+d/2} bxdx = bd.D^2 \tag{37}$$

It is clear that the effect of the articulation is to remove the $bd^3/12$ term from the second moment of waterplane area. For large D (ie D $\gg$ d), this term is small and does not influence the value of the overall second moment of area unduly.

Similarly for the circular waterplane area, of radius r at a distance D from the vessel axis, the rigid and articulated results are

$$I_{ry} = \int_{A_w} x^2 dA_w = \frac{\pi r^4}{4} + \pi r^2 . D^2 \tag{38}$$

and

$$I_{ay} = \pi r^2 . D^2 \tag{39}$$

Then for the rectangular waterplane area

$$\frac{I_{ay}}{I_{ry}} = \frac{1}{1 + \frac{d^2}{12D^2}} \tag{40}$$

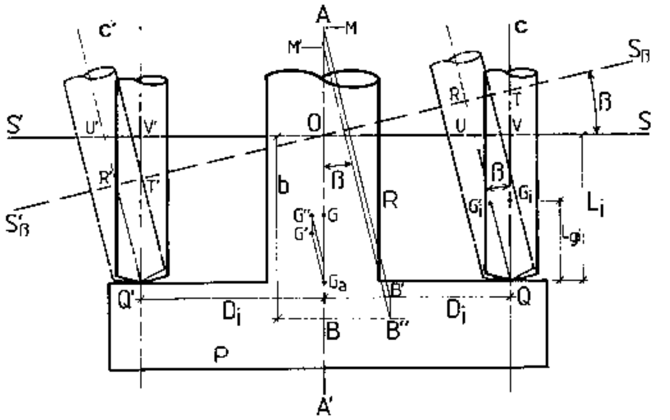

Fig.4. The Semiflex with Articulated Columns

and for the circular waterplane area

$$\frac{I_{ay}}{I_{ry}} = \frac{1}{1 + \frac{r^2}{4D^2}} \tag{41}$$

Fig.3 shows a plot of I_{ay}/I_{ry} as a function of D/d and D/2r to illustrate the strong influence of this ratio on the second moment of area ratio. Note that equation (35) contains terms due to shifts in the centres of gravity and buoyancy for the articulations. These cause a net reduction in stability which is much greater than the reductions shown in Fig.3 due to second moments of articulation waterplane areas.

3. The Theory for Large Angles of Inclination

This theory is developed using the specific hull shape of the semisubmersible vessel described in Fig.1. The rigid body case is considered first and generalised through to the articulated case.

3.1 The Rigid Body Equations

Consider view YY for the semisubmersible hull shape shown in Fig.1. Fig.4 displays a section through one of the three column pairs about the axis XX′ shown in Fig.1. The supporting structure connecting the buoyancy chambers B to the deck is represented in Fig.4 by the central rigid body R. A submerged pontoon P and circular rigid cylindrical columns C and C′ are disposed symmetrically about axis AA′. The ith such column pair shown in Fig.4 has column radii r_i, waterplane area A_i ($= \pi r_i^2$) at a distance D_i from axis AA′. SS′ is the still waterplane when the vessel is on an even keel whereas $S_\beta S'_\beta$ is the waterplane after a vessel inclination through angle β. B and B′ are the vessel centres of buoyancy before and after the inclination with BB″ and B″B′ being the corresponding displacements perpendicular and parallel to the vessel centre plane respectively.

Then, BB″ is given by considering the horizontal change in moment about an axis through O perpendicular to the plane of the diagram from the submerging and emerging volumes on the central waterplane area due to body R and the rigid columns C, C′. Using conventional hydrostatic analysis for large angles of β it can readily be shown that

$$BB'' = \frac{1}{V}[I_R + \sum_i I_i] \tan\beta \tag{42}$$

where I_R is the second moment of waterplane area of the centre body R. I_i is the second moment of waterplane area of the pair of columns about an axis through O perpendicular to the plane of the diagram and is of the form

$$I_i = 2\left[\frac{\pi r_i^4}{4} + \pi r_i^2 . D_i^2\right] \tag{43}$$

for each pair.

Similarly the distance B″B′ is obtained by taking moments of the submerging and emerging volumes about an axis through O perpendicular to the plane of the diagram. This gives

$$B''B' = \frac{1}{2V}[I_R + \sum_i I_i]\tan^2\beta$$

Since

$$BM = \frac{BB''}{\tan\beta} + B'B'' \quad (45)$$

$$BM = \frac{[I_R + \sum_i I_i]}{V} + \frac{\frac{1}{2}[I_R + \sum_i I_i]\tan^2\beta}{V} \quad (46)$$

This relationship is analogous to the 'wall sided' formula for ship sections. The value of BM above can be combined with BG to compute a metacentric height or righting moment arm (GZ) for each inclination.

3.2 The Influence of Articulated Appendages

For the case when columns C and C′ are articulated at Q and Q′ (see Fig. 4), an inclination β will induce a rotation of angle β in each of the columns so that they, in fact, remain vertical with their axes at right angles to the waterplane. Fig. 4 illustrates the new positions of the articulated columns perpendicular to the inclined vessel waterline $S_\beta S'_\beta$.

Now

$$UV = l_i\tan\beta$$

and

$$OU = D_i - l_i\tan\beta$$

Therefore

$$OR = (D_i - l_i\tan\beta)\cos\beta \quad (47)$$

Also

$$QT = l_i + D_i\tan\beta$$

Therefore

$$QR = (l_i + D_i\tan\beta)\cos\beta$$

so that

$$QR - QV = (l_i + D_i\tan\beta)\cos\beta - l_i$$

Similarly,

$$U'V' = l_i\tan\beta$$

$$OU' = D_i + l_i\tan\beta \quad (48)$$

$$OR' = (D_i + l_i\tan\beta)\cos\beta$$

and

$$Q'V' - Q'R' = l_i - (l_i - D_i\tan\beta)\cos\beta$$

As the vessel shown in Fig. 4 inclines, the submerged volume deficit between zero inclination and angle β is given by

$$A_i[U'R' - UR] = 2A_il_i\sin\beta\tan\beta$$

for each column pair. The total volume deficit is, therefore,

$$\sum_i A_il_i\sin\beta\tan\beta$$

and the parallel sinkage is

$$p = \frac{\sum_i l_iA_i}{A_W + \sum_i A_i}\sin\beta\tan\beta = k\sin\beta\tan\beta$$

where

$$k = \frac{\sum_i l_iA_i}{A_W + \sum_i A_i}$$

Taking moments of submerging and emerging volumes about an axis through O perpendicular to the plane of the diagram gives

$$V\,.\,BB'' = I_R\tan\beta + \tfrac{1}{2}\sum_i \pi r_i^2[(QR - QV)\{OR\cos\beta + \frac{(QR - QV)}{2}\sin\beta\} + (Q'V' - Q'R')\{OR'\cos\beta + \frac{(Q'V' - Q'R')}{2}\sin\beta\}] - \sum_i 2l_{bi}V_i\sin\beta - \sum_i 2pA_il_i\tan\beta\cos^2\beta \quad (49)$$

using volume integrations which are not included here.

Thus

$$V\,.\,BB'' = I_R\tan\beta + \tfrac{1}{2}\sum_i \pi r_i^2[\{(l_i + D_i\tan\beta)\cos\beta - l_i\}\{(D_i - l_i\tan\beta)\cos^2\beta + \tfrac{1}{2}\langle(l_i + D_i\tan\beta)\cos\beta - l_i\rangle\sin\beta\} + \{l_i - (l_i - D_i\tan\beta)\cos\beta\}\{(D_i + l_i\tan\beta)\cos^2\beta + \tfrac{1}{2}\langle l_i - (l_i - D_i\tan\beta)\cos\beta\rangle\sin\beta\} - 2l_{bi}l_i\sin\beta - 2pl_i\tan\beta\cos^2\beta]$$

after multiplying out and simplifying, this reduces to

$$BB'' = \frac{1}{V}[I_R\tan\beta + \tfrac{1}{2}\sum_i \pi r_i^2\{l_i^2\sin^3\beta + D_i^2\sin\beta(1 + \cos^2\beta) - 2l_{bi}l_i\sin\beta - 2pl_i\tan\beta\cos^2\beta\}] \quad (50)$$

Similarly by taking moments of volume about an axis through O perpendicular to the plane of the diagram and integrating we get

$$V\,.\,B''B' = \tfrac{1}{2}I_R\tan^2\beta + \tfrac{1}{2}\sum_i \pi r_i^2[(QR - QV)\{b + OR\sin\beta - \tfrac{1}{2}(QR - QV)\cos\beta\} - (Q'V' - Q'R')\{b + OR'\sin\beta + \tfrac{1}{2}(Q'V' - Q'R')\cos\beta\}] - \sum_i 2l_{bi}V_i(1 - \cos\beta) + \sum_i 2pA_i[b - l_i\sin^2\beta + \frac{p}{2}\cos\beta] \quad (51)$$

with b denoting the distance OB shown in Fig. 7.

Therefore

$$V\,B''B' = \tfrac{1}{2}I_R\tan^2\beta + \tfrac{1}{2}\sum_i \pi r_i^2[\{(l_i + D_i\tan\beta)\cos\beta - l_i\}\{b + (D_i - l_i\tan\beta)\cos\beta\sin\beta - \tfrac{1}{2}\langle(l_i + D_i\tan\beta)\cos\beta - l_i\rangle\cos\beta\} - \{l_i - (l_i - D_i\tan\beta)\cos\beta\}\{b - (D_i + l_i\tan\beta)\cos\beta\sin\beta + \tfrac{1}{2}\langle l_i - (l_i - D_i\tan\beta)\cos\beta\rangle\cos\beta - 2l_{bi}l_i(1 - \cos\beta) + 2p(b - l_i\sin^2\beta + \frac{p}{2}\cos\beta)] \quad (52)$$

Again after multiplying out and simplifying, we get

$$V\,.\,B''B' = \tfrac{1}{2}I_R\tan^2\beta + \tfrac{1}{2}\sum_i \pi r_i^2[(D_i^2 - l_i^2)\sin^2\beta\cos\beta + 2l_i^2(1 - \cos\beta) - 2bl_i + 2bl_i\cos\beta - 2l_{bi}l_i(1 - \cos\beta) + 2p(b - l_i\sin^2\beta + \frac{p}{2}\cos\beta)] \quad (53)$$

Then

$$BM = \frac{BB''}{\tan\beta} + B'B''$$

$$= \frac{1}{V}[I_R + \tfrac{1}{2}I_R\tan^2\beta] + \frac{1}{2V}\sum_i \pi r_i^2[l_i^2\sin^2\beta\cos\beta$$

$$+ D_i^2\cos\beta\,(1+\cos^2\beta) + (D_i^2 - l_i^2)\sin^2\beta\cos\beta$$

$$+2l_i^2(1-\cos\beta) - 2bl_i + 2bl_i\cos\beta - 2l_{bi}l_i$$

$$- 2l_ik\sin^2\beta\cos\beta + 2k\sin\beta\tan\beta(b - l_i\sin^2\beta$$

$$+ \tfrac{1}{2}k\sin^2\beta)] \qquad (54)$$

$$= \frac{1}{V}[I_R + \tfrac{1}{2}I_R\tan^2\beta]$$

$$+ \frac{1}{V}\sum_i A_i[D_i^2\cos\beta + l_i(l_i - b)(1-\cos\beta)$$

$$- l_{bi}l_i - l_ik\sin^2\beta\cos\beta$$

$$+ k\sin\beta\tan\beta(b - l_i\sin^2\beta + \tfrac{1}{2}k\sin^2\beta)] \qquad (55)$$

An identical result can be obtained if the moments of volumes are taken about an axis through B and perpendicular to plane $S_\beta S_\beta'$. The expression in equation (55) shows the contributions of the rigid central waterplane cutting member (with second moment of area, I_R) and of the articulated columns such as C and C′. The expression is equivalent to the small angle result of equation (27) when $\beta \to 0$.

It is interesting to note the effects of distances l_i and b on the large angle hydrostatics. The $D_i^2\cos\beta$ term is generally dominant for large D_i but the additional modification to BM at high angles is eliminated if $l_i = b$. Furthermore this modification changes sign depending on $l_i > b$ or $l_i < b$.

The $l_{bi}l_i$ term causes a constant and significant reduction in BM due to movement of the articulation submerged volumes whereas the terms due to parallel sinkage change the BM values by small amounts.

The movement of the position of the centre of gravity due to the articulated columns must be considered for its effect in increasing the metacentric height. For the large angle problem, Fig. 4 illustrates the horizontal shift of G to G″ together with a vertical downward shift to G′.

The effective increase in metacentric height then is

$$GG_a = \frac{GG''}{\tan\beta} + G'G'' \qquad (56)$$

The effects of each pair of columns i can be summed to give a total horizontal shift of the centre of gravity as

$$GG' = \frac{\sum_i m_i l_{gi}(1-\cos\beta)}{M} \qquad (57)$$

where m_i is the mass of each of the columns and l_{gi} is the vertical distance between the column centre of gravity and the point of articulation at zero angle of inclination. M is the total vessel mass.

In a similar manner, the vertical shift in the centre of gravity position is obtained as

$$G'G'' = \frac{\sum_i m_i l_{gi}(1-\cos\beta)}{M}. \qquad (58)$$

Then the net increment in GM is obtained via equation (56) as

$$GG_a = \frac{\sum_i m_i l_{gi}}{M} \qquad (59)$$

with the contributions of $\cos\beta$ cancelling out. The total metacentric height can then be

$$G_aM_y = BM_y - BG + GG_a$$

$$= \frac{1}{V}[I_R + \tfrac{1}{2}I_R\tan^2\beta] + \frac{1}{V}\sum_i A_i[D_i^2\cos\beta$$

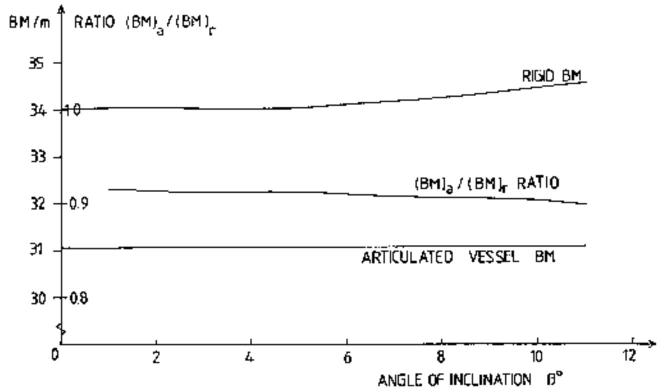

Fig. 5. Distances BM as Functions of Angle of Inclination β

$$+ l_i(l_i - b)(1-\cos\beta) - l_{bi}l_i - l_ik\sin^2\beta\cos\beta$$

$$+ k\sin\beta\tan\beta(b - l_i\sin^2\beta + \tfrac{1}{2}k\sin^2\beta)]$$

$$+ \frac{\sum_i m_i l_{gi}}{M} - BG \qquad (60)$$

3.3 Numerical Results

The influence of articulations on the hydrostatics of a floating body is illustrated by numerical values for an articulated column pair and submerged pontoon arrangement shown in Fig. 4. Numerical values of $D_i = 64{\cdot}95$ m, $l_i = 42$ m, $b = 31{\cdot}1$ m, $r_i = 3{\cdot}5$ m are taken from data for a proposed vessel (Fig. 1) of 30,750 tonnes displacement. Only the BM contribution due to the articulated columns is considered first to highlight differences between the articulated and rigid body hydrostatics.

Fig. 5 displays the variation of BM as a function of angle of inclination, β. The rigid equivalent value is compared with the value arising from the articulated columns.

The BM contribution from the rigid column arises in the conventionally accepted manner whereas the BM contribution of the articulated columns starts from a lower value due to the net effect of the shift in total vessel centres of buoyancy due to articulation rotation. This feature is not as much of a drawback as it appears at first sight because the existence of articulations allows the values of D_i to be large without significant structural penalties and therefore, the base BM (for $\beta = 0°$) has a large value.

Fig. 5 also displays the ratio of articulated to rigid BM value which tends to fall off more rapidly with angle due to the increase in rigid column BM with angle of inclination.

Fig. 6 displays the variation in righting moment arm GZ with angle of inclination β. These curves include the effective increase in metacentric height which occurs in the presence of the articulations. Despite this feature, the articulated

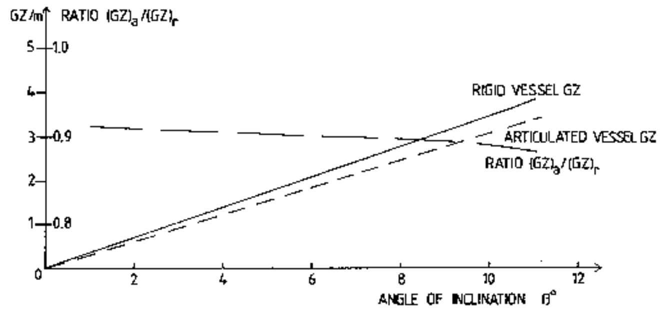

Fig. 6. Righting Moment Arms as Functions of Angle of Inclination β

columns contribute a smaller righting moment arm than the equivalent rigid columns. Fig. 6 also displays the ratio of articulated to equivalent rigid body GZ value. Values of $m_i = 657$ tonnes and $l_{gi} = 21$ m are assumed for each articulated column.

4. SPECIAL CONSIDERATIONS FOR VESSELS WITH ARTICULATED APPENDAGES

There are a number of additional features that must be accounted for during the assessment of the hydrostatics of a vessel with articulated appendages, particularly from the point of view of classification society certification regulations. These features and their likely impact on certification procedures are summarised below.

4.1 Forces at the Articulation Joint for Large Angles of Inclination

The presence of excess buoyancy over and above the weight of an individual articulation ensures that the member remains vertical during vessel inclinations and that a tensile force is maintained in the articulation joint. During vessel inclination, further submergence of an articulated member increases the member buoyancy and the consequent internal joint tension. However, the corresponding articulated member which would be emerging on the opposite beam of the vessel experiences a reduction in buoyancy force accompanied by a reduction in the joint tension.

This variation of joint tension with vessel inclination raises two points; the first being that the vessel designer must ensure adequate angular stability and joint tension for articulated members emerging above their still water position due to large angles of vessel inclination. Alternatively, an articulated member can be permitted to go 'unstable' such that it will no longer remain vertical but will take up an equilibrium position at some angle to the still waterplane. It can readily be shown that in this mode, the joint tension will remain positive and constant such that it is independent of variation in the vertical distance of the articulated joint below still water level. In this case, the high angle hydrostatic calculation must account for the change over of a member from the stable upright mode to the tilted mode at high emerged distances.

4.2 Damaged Stability Due to Articulation Failure

In addition to the conventional design-related wind heeling moment as well as intact and damaged stability requirements, a vessel with articulated members must be assessed for the effects of articulation failure. Each articulated joint can be envisaged to be a mechanical universal joint typical of many that are in current use for offshore oil loading towers. These joints are generally designed with a very large factor of safety such that failure of the primary vessel structure is likely to precede failure of a joint. Despite this, a damaged stability calculation needs to be performed by postulating the complete loss of one of the several articulated members that would be used on the type of vessel shown in Fig. 1.

4.3 Calculation of Wind Heeling Moments

The conventional hydrostatic stability requirement that the hydrostatic righting moment must exceed the wind heeling moment by an adequate margin remains largely unaffected by the articulations except for one compensating factor. Wind forces on the articulated members above still water level will not be transmitted directly as wind heeling moments into the base structure. An equilibrium force consideration will readily show that the base structure will experience a force at the articulation which is equal to the total wind force and in the same direction. This shift in the point of force application will reduce the total overturning moment on the structure which together with the high metacentric heights inherent in this design allow conventional wind heeling moment requirements to be satisfied with a substantial margin.

5. DISCUSSION AND CONCLUSIONS

The extension of the theory of conventional rigid body hydrostatic stability to non-rigid bodies with articulated members shows that the presence of the articulations reduces the stability by a relatively small proportion for articulated member waterplane areas that are well separated from the principal axes of the waterplane. The stability reduction is primarily due to the movements of the centres of buoyancy and gravity induced by rotation of the articulations. Consideration of the effects of articulations on damage stability criteria and wind heeling moments indicates that the articulations introduce no significant difficulties for classification society certification requirements of ensuring adequate hydrostatic stability to the same standards as demanded for conventional semisubmersible vessels.

The design of conventional semisubmersible vessels is a compromise of conflicting requirements for low motion response to waves, high deck payload and low structural steel weight. The low motion response is generally obtained at the expense of inadequate deck payload such that this is usually an operational limitation. The design of a new generation of semisubmersibles with articulated columns, as shown in Fig. 1, breaks the hydrostatic and hydrodynamic conflict between low motion response and high deck payload. The placing of waterplane area at substantial distances from the vessel principal axes without a structural penalty allows high deck payload via the high metacentric height while maintaining low waterplane area for a low wave induced motion response.

REFERENCE

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The authors gratefully acknowledge the support of Mr J.G. Cluff of Cluff Oil Ltd and Mr A.G. Copson of Cluff Copson Designs Ltd towards the work reported in this paper. We wish to thank Mr A.G. Copson for permission to reproduce a general arrangement drawing of the Cluff Copson Semiflex semisubmersible design (Fig. 1).

WRITTEN DISCUSSION

Professor J. P. Kundu, B.E., B.Tech. (Member): It seems the caption for Fig. 3 is not what it should have been. Perhaps 'Ratios of Second Moments of Articulated to Rigid Waterplane Areas' would have been ideal. While deriving rigid body equations in Section 3.1 and also elsewhere, reference has been made to the columns in Fig. 4 by C and C^1. Actually however, C and C^1 have not been printed by the side of the columns. Again L_1 in the same figure has been referred to as l_1 in all deductions that follow. Is this just a minor discrepancy and acceptable?

In the upright condition ($\beta = 0°$), GZ for both articulated and rigid vessels being zero, the ratio $\frac{GZ_a}{GZ_r} = \frac{0}{0}$, i.e. indeterminate. The curve of this ratio, however, has been shown to originate from a value 1·08 (Fig. 6). Perhaps it should have started just a bit earlier from the y-axis.

The authors have stated that investigation into the effect of the complete loss of one of the several articulated towers is required. A similar study of the effect of a damaged/flooded tower on the articulation joint and also on the platform as a whole is also desired. It is likely that being articulated, the flooded tower would heel arbitrarily depending upon wave directions and this would cause severe strain on the articulation and also disturb the stability of the platform as a whole.

Before towing the platform from its builder's yard to the site of operation, perhaps a robust arrangement for fixing the articulated towers in the vertical position is necessary. Otherwise while in motion, the towers are likely to become inclined in arbitrary directions and thereby adversely affect the platform's directional stability and also the towing operation as a whole.

The authors have presented a new approach for enhancing the deck payload of offshore vehicles. The subject matter is surely interesting and useful to students and teachers of Naval Architecture in general and to the designers of semi-submersibles in particular.

My hearty congratulations to the authors.

Dr D. B. McIver: The articulated vessel which the authors deal with is a clear departure from conventional semi-submersibles. Orthodox methods of studying its hydrostatic characteristics, applied readily to rigid structures, should therefore be viewed with some caution.

One important feature of the derivation presented for rigid bodies which is not explicitly stated is that the wetted surface of the body $z = z(x, y)$ is defined in body axes which rotate with the body. However, the function $z = z_{ai}(x, y)$ which defines the wetted surface of the articulated appendage in its initial equilibrium position can no longer do so when the semi-submersible is rotated and the articulated appendage has moved relative to the rest of the vessel. This has not been recognised by the authors, leading to an error in their derivation of the metacentric height of the platform.

Expressed in another way, the relative movement of the different parts of the vessel results in an inherent shift in the centre of buoyancy position analogous to that of the centre of gravity, the effect of which *is* included in the authors' derivation.

It would be too lengthy for this discussion to parallel their derivation by including additional sets of axes for the articulated appendages. Instead, a simpler approach is adopted.

Let V_r and V_a be the submerged volumes of the rigid and articulated parts of the structure respectively. $\bar{x}_r, \bar{y}_r, \bar{z}_r$ and x_{ai}, y_{ai}, z_{ai} are the co-ordinates of their centres of buoyancy in the initial equilibrium configuration.

After a rotation β about the y-axis, the net hydrostatic moment M_H about the y-axis through the centre of flotation F is:

$$M_H = \rho g V_r \bar{z}_r \beta - \rho g \int_{A_w} x^2 dA_w \beta + \sum_i \rho g V_{ai}(x_i + z_i \beta) - \sum_i \rho g x_i \int_{A_i} x dA_i \beta \quad (61)$$

Notice that the moment arm of the articulated appendage changes from x_i to $x_i + z_i\beta$ after the rotation. Assuming that they are symmetrically disposed about the y-axis, then after some substitution

$$M_H = -\rho g[I_{yy} + I_{ay} - V\bar{z} - \sum_i V_{ai}(z_i - \bar{z}_{ai})]\beta \quad (62)$$

where V is the net submerged volume, $\overline{Z}$ is the vertical co-ordinate of the vessel's centre of buoyancy and $\bar{z}_{ai}$ that of the i'th articulated appendage.

The gravitational moment is:

$$M_G = -(MgFG + \Sigma m_i g l_{gi})\beta \quad (63)$$

$$= -MgFG_a\beta \quad (64)$$

where the second term in equation (63) is the modification due to the articulations.

Noting that for vertical equilibrium

$$Mg = \rho g V \quad (65)$$

the nett moment about F may be written as

$$M_H + M_G = -[\rho g(I_{yy} + I_{ay}) - \rho g V BG + \sum_i m_i g l_{gi} - \rho g \sum_i V_{ai}(z_i - \bar{z}_{ai})] \quad (66)$$

$$= -\rho g V G_a M_y \quad (67)$$

The metacentric height is therefore

$$G_a M_y = \frac{I_{yy} + I_{ay}}{V} - BG + \sum_i \frac{(m_i l_{gi} - \rho V_{ai} l_{hi})}{\rho V} \quad (68)$$

where $l_{hi} = z_i - \bar{z}_{ai}$ is the height of the centre of buoyancy of the appendage above the articulation. The last term on the right of equation (68) is the correction due to the relative movement of the centre of buoyancy of the appendage. The inclusion of this additional effect of the articulation is therefore to reduce the metacentric height and therefore the margin of static stability from that presented in the paper. The net effect over that for a completely rigid structure depends on the sign of the quantity

$$m_i l_{gi} - \rho V_{ai} l_{hi} \quad (69)$$

for each appendage. This should be negative for small angle static stability of the appendage which in turn means that the metacentric height for the articulated platform is less than for a wholly rigid platform of the same configuration. This is what should be expected since the effect of the articulation is analogous to having a free-surface within the system.

Using the authors' dimensions for their Fig. 4, the metacentric height is 2·26 m *less* than that predicted in the paper (approximately 11·5 m). The net *reduction* in metacentric height due to the articulation is 1·37 m.

It appears the same omission occurs in the large displacement analysis.

AUTHORS' REPLY

The authors are grateful to Professor Kundu for his comments. The typographical errors that were pointed out have been corrected.

The effects of the complete loss of one of the articulated towers and the articulation joint tension variations due to a flooded tower are currently being examined in detail using both hydrodynamic analyses and model tests.

The towage of the platform from the builders yard to the operations site has been checked by model tests. These indicate that the towers incline by only a small amount (<3° from the vertical) during the towing operation and they do not exhibit untoward transverse oscillations due to vortex shedding effects. Thus, since the tower angles from the vertical are expected to be small, the consequential effects on platform stability and articulation joint tensions will be within acceptable bounds.

The authors are grateful to Dr McIver for pointing out the changes required in the analysis. These came to light independently shortly after the advance copy was released and have been incorporated into the final published version of the paper.