BOUNDARY ELEMENTS IN FLUID/STRUCTURE INTERACTION PROBLEMS

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ABSTRACT

The use of the finite element method for structural analysis is now commonplace. In the analysis of offshore structures parts of the structure are often in contact with a fluid either internally or externally. The fluid will affect the natural frequencies and mode shapes of the structure. In this paper a general purpose package of computer programs based on the boundary element method, for the solution of such problems is described. The specific example of the forced response of a pressure vessel containing a compressible fluid with a free surface is discussed as a worst case example. Simpler examples of situations incorporating only some of the features of this problem are also discussed.

INTRODUCTION

There have been a number of attempts to analyse fluid structure interaction problems using various techniques involving a mixture of finite element, boundary element and analytic representations for the fluid and structure, often the structure may be assumed to be rigid and inelastic, as in the case of diffraction problems, Brebbia and Walker (1978).

The first attempts at solving the fluid/structure interaction problems represented the fluid forcing on the structure analytically Miles (1956), Morison (1950). The motion of the structure however was not assumed to change the nature of the forcing appreciably although some attempts have been made to generalise the Morison equation to take into account this effect, Laird (1962), Brebbia and Walker (1979). The resulting approach has often involved a cyclic iterative procedure.

The next stage was to consider the fluid as three dimensional finite elements joined at the structural nodes to the structure's finite element mesh. For the external problem this however involves the use of some kind of asymptotic matching procedure (as in Chen and Mei (1974), Boreel (1974)) or the use of, so called, infinite finite elements. Another alternative is to use the Sommerfeld radiation condition on the boundary of the finite element mesh (Walker & Brebbia 1978). The use of finite elements for the fluid, and some kind of matching on the boundary of the finite element mesh has perhaps been superceded for most problems by the implementation of the Boundary Element method which has three important advantages.

1) The representation of the fluid is effected using two dimensional surface boundary elements which can be directly identified with the finite elements on the surface of contact with the structure

2) With careful choice of the fundamental solution infinite fluid regions may be represented in the same way.

3) For the internal problem (e.g. pressure vessels) a free surface boundary condition may be used to eliminate all nodal unknowns on the liquid surface not in contact with the structure.

Two methods of formally joining Finite and Boundary Element regions are given in Brebbia and Walker (1978), the problem is further discussed in Shaw (1978). Suitable fundamental solutions for infinite and semi-infinite regions are given in Brebbia and Walker (1980).

These methods have the added advantage that no iterative procedure is involved in determining the back reaction of the structure on the fluid.

DEFINITION OF THE PROBLEM

The problem to be solved is to determine the forced response of the vessel shown in Figure 1.

The compressible liquid comes up to the level indicated and is agitated internally by two Rushton impellers which rotate at a fixed speed. The convective velocity of the liquid is well subsonic so the acoustic field is essentially decoupled from the convective field which gives rise to a regular and (assumed known)forcing on the structure.

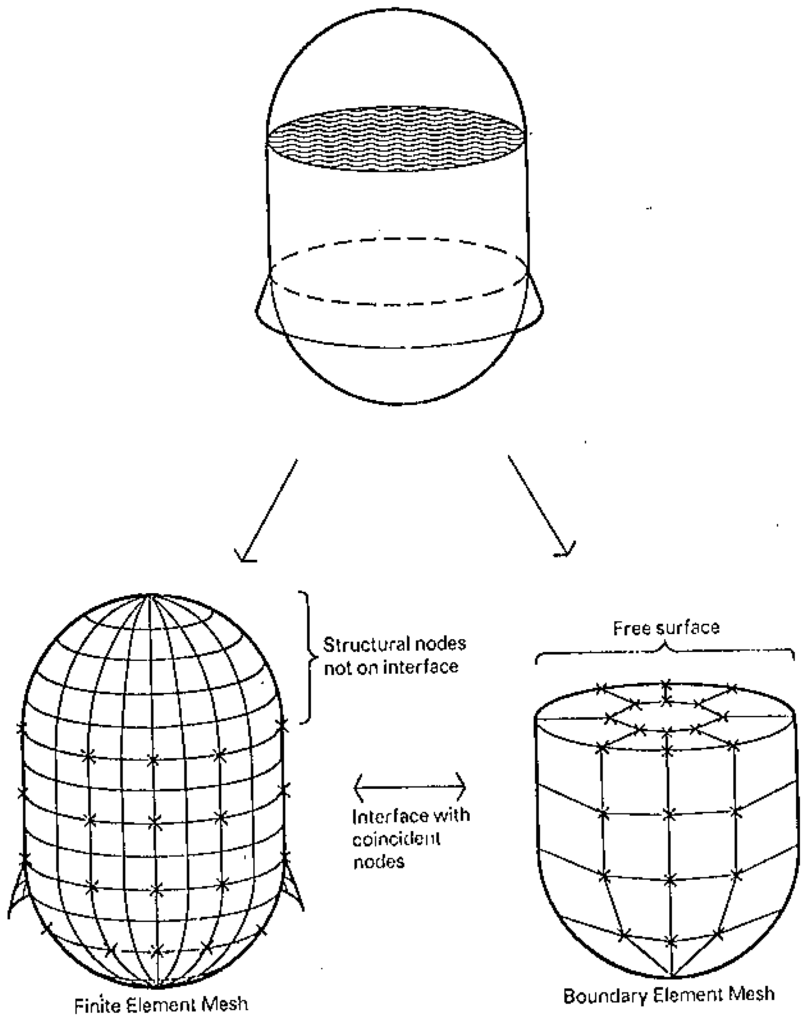

Figure 1 Idealisation of Fluid/Structure System for pressure vessel

Because the internal dimensions of the vessel correspond to about one wavelength at the frequency of agitation, we are near acoustic resonance and the acoustic field is the main mechanism of energy transfer through the fluid.

The procedure for the solution of fluid/structure interaction problems is shown in Figure 2. Figure 1 gives some idea of the geometry of the problem considered.

BOUNDARY ELEMENT FORMULATION (FLUID REGION)

In this section we look at the application of the B.E.M. to the three-dimensional acoustic field problem. For the acoustic field within the fluid we may define a velocity potential ϕ by:-

$$\underline{u} = \nabla \phi = \text{Grad}\phi \quad \text{.......(1)}$$

Where $\underline{u}$ is the (Eulerian) velocity of the fluid. The governing equation of the fluid may now be written:-

$$\nabla^2 \phi = \frac{1}{C^2} \frac{\partial^2 \phi}{\partial t^2} \quad \text{.......(2)}$$

Assuming a harmonic time dependence for ϕ of the form $e^{i\omega t}$, (2) becomes:-

$$\nabla^2 \phi + \kappa^2 \phi = 0 \quad \text{.......(3)}$$

$$\text{where } \kappa = \frac{\omega}{c} \quad \text{.......(4)}$$

Or more generally, if we have a distribution $m(\underline{r})$ of sources (with $\underline{r}$ the position vector) we may write

$$\nabla^2 \phi + \kappa^2 \phi = m(\underline{r}) \quad \text{.......(5)}$$

consider $\nabla^2 G + \kappa^2 G = \delta(\underline{r} - \underline{r}')$ ....(6)

where G is the free space Green's function or fundamental solution, which is a function of two variables the source point $\underline{r}'$ and the observation point $\underline{r}$ (Figure 3); δ is the Dirac delta function.

The solution of equation (6) is given explicitly by:-

$$G(r, r') = -\frac{e^{-i\kappa r}}{4 \pi r} \quad \text{.......(7)}$$

where $r = |\underline{r} - \underline{r}'|$

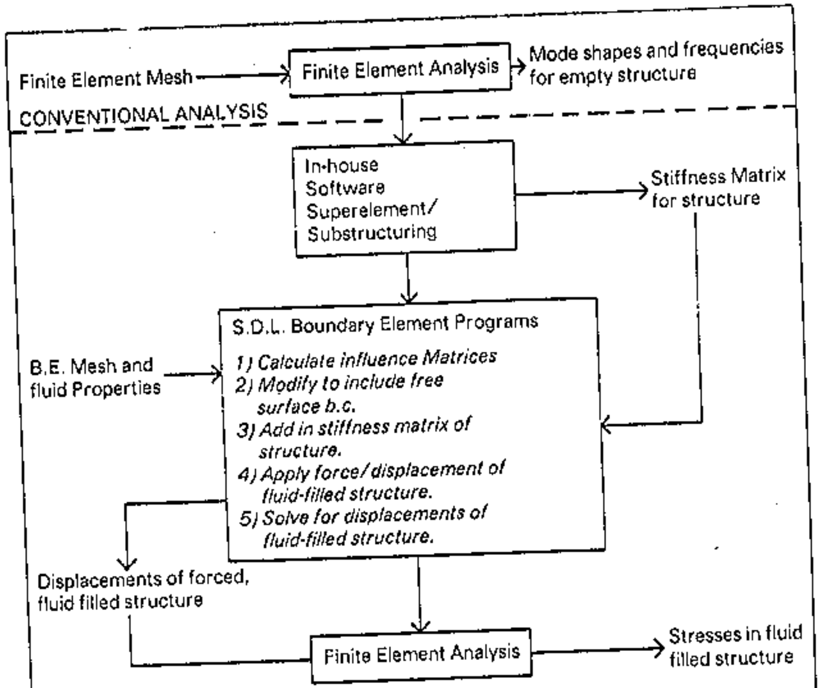

Figure 2 Solution Procedure Fluid/Structure Interaction

Green's theorem states, that for any two functions ϕ and ψ which are sufficiently differentiable for the ∇^2 to exist we may write

$$\int_V \{\psi \nabla^2 \phi - \phi \nabla^2 \psi\} dV = \int_S \left\{\psi \frac{\partial \phi}{\partial n} - \phi \frac{\partial \psi}{\partial n}\right\} dS \qquad \ldots\ldots\ldots (8)$$

where the regions of integration are indicated in Figure 4 and n is the outward normal to V.

If we choose ψ to be the fundamental solution G, then using (6) and the selective property of the delta function we can eliminate the volume integral to obtain:-

$$\phi(r') = -\int_S \left\{G(\underline{r}, \underline{r}') \frac{\partial \phi}{\partial n} - \phi \frac{\partial G(\underline{r}, \underline{r}')}{\partial n}\right\} dS(\underline{r}) \qquad \ldots (9)$$

This is an identity for all $\underline{r}'$ in V.

In our case we have no net creation or destruction of fluid in our problem region and we have put $m(\underline{r}) \equiv 0$. If we now choose $\underline{r}'$ to be a point on the boundary S, then (9) becomes

$$c\, \phi(\underline{r}') = -\int_S G(\underline{r}', \underline{r}) \frac{\partial \phi}{\partial n} - \phi \frac{\partial G(\underline{r}', \underline{r})}{\partial n} dS(\underline{r}) \qquad \ldots (10)$$

$$\text{where } c = \frac{\alpha}{4\pi} \qquad \ldots (11)$$

and α is the solid angle interior to Γ at $\underline{r}'$ (usually 2π for a smooth surface).

We now discretise Γ into N flat "facets" or "elements" of area $A^{(j)}$, then $c = \frac{1}{2}$ (Figure 5).

We choose to use constant elements to avoid the corner problems here, we approximate ϕ over element j by its value at the centroid of the area $A^{(j)}$ say. Similarly put

$$\frac{\partial \phi}{\partial n} \approx \left[\frac{\partial \phi}{\partial n}\right]_j = q_j \qquad \ldots (12) \text{ say}$$

Equation 10 may then be written:-

$$\frac{\phi_j}{2} = -\sum_{j=1}^{N} \left\{\int_{S_j} G \frac{\partial \phi}{\partial n} dS_j - \int_{S_j} \phi \frac{\partial G}{\partial n} dS_j\right\} \qquad \ldots (13)$$

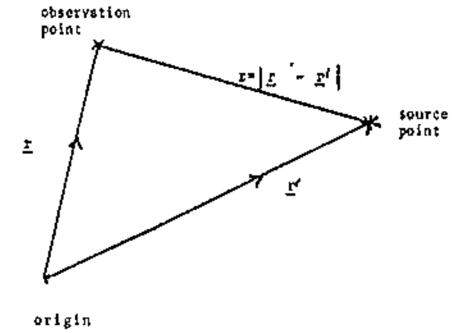

Figure 3 - Physical interpretation of Green's Function

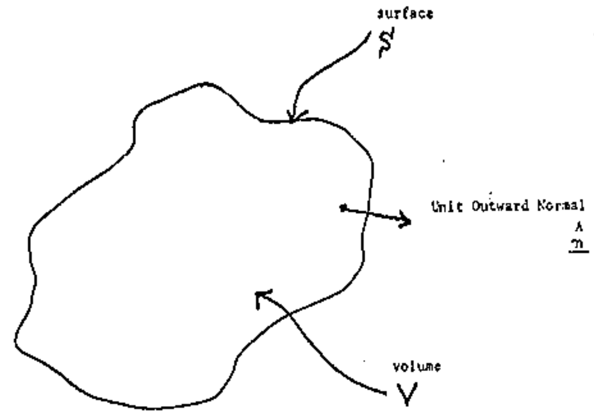

Figure 4 - Problem region for Green's Theorem

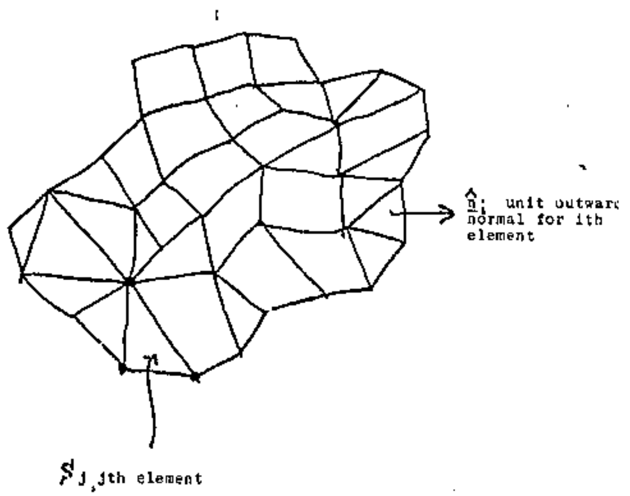

Figure 5 Discretised surface of problem region

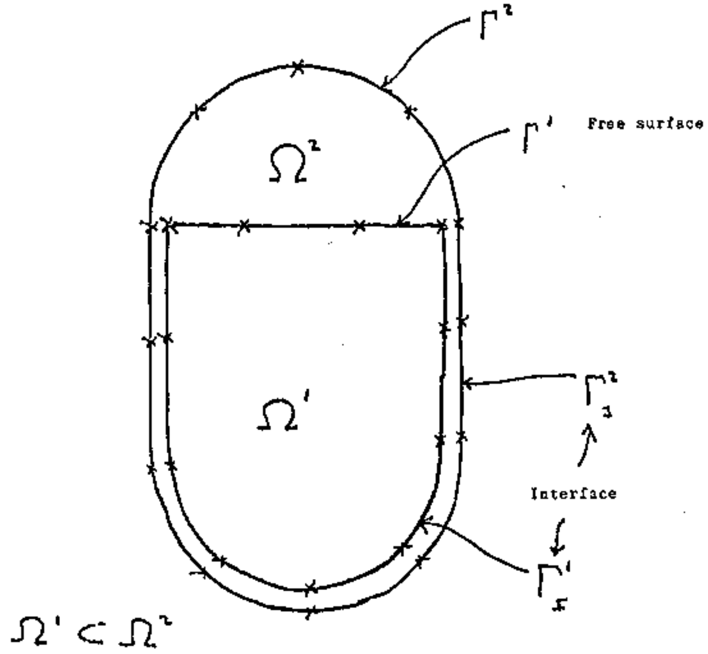

Figure 6 Section through problem region showing subsets of bounding surfaces

hence approximately

$$\frac{\phi_i}{2} = - \sum_{j=1}^{N} \left\{ q_j \int G \, dS_j - \phi_j \int \frac{\partial G}{\partial n} \, dS_j \right\} \quad \ldots. (14)$$

making the following changes in notation

replace G by g_{ij} where $\underline{r}_j = \underline{r}$, $\underline{r}_i = \underline{r}'$

$$g_{ij} = - \frac{-\exp\{-i\kappa|\underline{r}_j - \underline{r}_i|\}}{4\pi|\underline{r}_j - \underline{r}_i|} \quad \ldots. (15)$$

$$h_{ij} = \frac{\partial g_{ij}}{\partial n} \quad \ldots. (16)$$

Note that g_{ij} is symmetric in i and j

$$\text{Let } G_{ij} = A^{(j)} \int_{S_j} g_{ij} \, dS_j \quad \ldots. (17)$$

$$\text{and } \hat{H}_{ij} = A^{(j)} \int_{S_j} h_{ij} \, dS_j \quad \ldots. (18)$$

This choice of variable has been made to make G_{ij} nearly symmetric, this follows from the symmetry of g_{ij}.

Then equation (14) may be written:-

$$\frac{\phi_i}{2} = \sum_{j=1}^{N} \left\{ - G_{ij} \, q_j + H_{ij} \, \phi_j \right\} \frac{1}{A^{(j)}} \quad \ldots. (19)$$

$$\text{letting } H_{ij} = \hat{H}_{ij} - \frac{1}{2} \delta_{ij} A^{(j)} \quad \ldots. (20)$$

where δ_{ij} is the Kronecker delta defined by

$$\delta_{ij} = 0 \qquad i \neq j$$
$$\delta_{ij} = 1 \qquad i = j \quad \ldots. (21)$$

(19) may be more concisely written

$$\sum_{j=1}^{N} \frac{H_{ij}}{A^{(j)}} \phi_j = \sum_{j=1}^{N} \frac{G_{ij}}{A^{(j)}} q_j \quad \ldots. (22)$$

As we are considering harmonic time dependence we may write the normal velocity through element j in terms of the normal displacement as:-

$$q_j = i\omega\, U_j \qquad \text{.... (23)}$$

and using Bernoulli's equation we can write the velocity potential on element j in terms of the pressure or normal force F_j on that element.

$$\phi_j = -\frac{P_j}{i\omega\rho} = -\frac{F_j}{A^{(j)} i\omega\rho} \qquad \text{.... (24)}$$

where $A^{(j)}$ is the area of the j^{th} element and no summation is implied over j in (24).

Hence

$$\frac{H_{ij}}{(A^{(j)})^2} F_j = \omega^2\rho \frac{G_{ij}}{A^{(j)}} U_j \qquad \text{... (25)}$$

hence writing

$$\hat{H}_{ij} = H_{ij}/(A^{(j)})^2 \qquad \text{.... (26)}$$

and $\hat{G}_{ij} = G_{ij}/A^{(j)}$

In dyadic notation (25) becomes

$$\underline{F} = \omega^2\rho\, \underline{\underline{\hat{H}}}^{-1}\, \underline{\underline{\hat{G}}}\, \underline{U} \qquad \text{.... (27)}$$

Let $\underline{\underline{A}} = \omega^2\rho\, \underline{\underline{\hat{H}}}^{-1}\, \underline{\underline{\hat{G}}}$ .... (28)

then $\underline{F} = \underline{\underline{A}}\, \underline{U}$ .... (29)

Equation (29) is of the same form as the finite element 'stiffness' formulation linking forces and displacements.

The integrals G_{ij} and H_{ij} were evaluated numerically using four point Gaussian integration, i.e.

Sixteen points were used on each element (Stroud & Secrest 1966).

Note

1. $\hat{H}_{ii} = 0$ as it represents the integral of the flux normal to the element from a point source in the plane of the element. This singular term need not therefore be evaluated explicitly.

2. In the incompressible limit i.e. $c \to \infty$ and $\kappa \to o$ we have the usual potential theory formulation, H and G being independent of frequency. The matrix A then has the form of a mass or "added mass" matrix as we would expect. For convenience, in this paper the term stiffness matrix will be assumed to incorporate the mass matrix in the form $\underline{\underline{K}} - \omega^2\underline{\underline{M}}$ where it exists.

APPLICATION OF THE FREE SURFACE BOUNDARY CONDITION

On the free surface of a compressible fluid we have

$$\frac{\partial\phi}{\partial z} = \frac{\omega^2\phi}{g} \qquad \text{.... (30)}$$

for harmonic time dependence, which becomes after discretisation

$$P_j = g\rho A^{(j)}\, U_j \qquad \text{.... (31)}$$

We denote free surface values of the variables by a subscript 0, then:-

$$F_{0_j} = g\rho U_{0_j}\, A^{(j)} \qquad \text{.... (32)}$$

From above

$$\underline{F} = \underline{\underline{A}}\, \underline{U} \qquad \text{.... (33)}$$

or in partitioned form

$$\begin{bmatrix} \underline{F} \\ \underline{F}_0 \end{bmatrix} = \begin{bmatrix} \underline{\underline{A}}_{11} & \underline{\underline{A}}_{10} \\ \underline{\underline{A}}_{01} & \underline{\underline{A}}_{00} \end{bmatrix} \cdot \begin{bmatrix} \underline{U} \\ \underline{U}_0 \end{bmatrix} \qquad \text{.... (34)}$$

$$\therefore\ \underline{F}_0 = \underline{\underline{A}}_{01}\, \underline{U} + \underline{\underline{A}}_{00}\, \underline{U}_0 = \rho g\, \{A^{(j)}\, U_{0j}\} \qquad \text{.... (35)}$$

$$\therefore\ \underline{\underline{A}}_{01}\, \underline{U} = \left[-\underline{\underline{A}}_{00} + \rho g A^{(j)}\, \underline{\underline{I}}\right] \underline{U}_0 \qquad \text{.... (36)}$$

or $\underline{U}_0 = \left[-\underline{\underline{A}}_{00} + \rho g A^{(j)}\, \underline{\underline{I}}\right]^{-1} \underline{\underline{A}}_{01}\, \underline{U}$

but $\underline{F} = \underline{\underline{A}}_{11}\underline{U} + \underline{\underline{A}}_{10}\underline{U}_0 = \left[\underline{\underline{A}}_{11} + \underline{\underline{A}}_{10}(-\underline{\underline{A}}_{00} + \rho g A^{(j)}\, \underline{\underline{I}})^{-1} . \underline{\underline{A}}_{01}\right]\underline{U}$

.... (37)

$$\underline{F} = \underline{\underline{A}}'\, \underline{U} \qquad \text{.... (38)}$$

We have now expressed the forces at nodes on the fluid/structure interface in terms of the displacements at these nodes only. $\underline{\underline{A}}'$ is the modified matrix after the application of the free

JOINING OF FINITE AND BOUNDARY ELEMENT REGIONS

Consider the discretised problem depicted in Figure 6 divided into two regions Ω^2, Ω^1. Ω^1 is the fluid region.

The two regions are bounded by surfaces Γ^1 and Γ^2 as follows:-

(i) Γ_s^1 that part of the surface of Ω^1 not in contact with the structure (the free surface)

(ii) Γ_I^1 that of Ω^1 in contact with the structure (the interface nodes)

(iii) Γ_I^2 that part of Ω^2 in contact with the fluid.

(iv) Γ_s^2 that part of Ω^2 not in contact with the fluid.

In Figure 6 the nodes and elements of the interfaces Γ_I^1 and Γ_I^2 are shown as two distinct surfaces for clarity. The nodal unknowns $\underline{F}$ and $\underline{U}$ are in fact related by two conditions on the interface.

(i) Compatibility, the nodal displacements at corresponding nodes are equal on the interface.

$$\underline{U}^1_I = \underline{U}^2_I = \underline{U}_I \qquad \ldots\ldots\ldots\ldots(39)$$

(ii) Equilibrium, the interfacial forces; fluid on structure, structure on fluid, must be equal and opposite.

$$\underline{F}_I^1 = -\underline{F}_I^2 = \underline{F}_I \qquad \ldots\ldots\ldots\ldots(40)$$

This is quite apart from any externally applied forces on the structure which we denote by $\underline{F}_e$. (The subscripts and superscripts on $\underline{F}$ and $\underline{U}$ denote those components of $\underline{F}$ and $\underline{U}$ corresponding to the various subsets of Γ with the same subscripts).

In Ω^1 we have obtained the normal forces at the mid-points we can either:-

1) Arrange the fluid and structure meshes so that mid-points in Ω^2 correspond to corner nodes of the mesh in Ω^2.

2) Modify the elements of $\underline{\underline{A}}$ (the stiffness matrix for Ω') so that $\underline{F}$ and $\underline{U}$ may be re-interpreted as nodal forces and displacements at the corner nodes in the three global co-ordinate directions. This was the approach used. The force at the mid-point was split equally among the corner nodes this gives the algorithm for the modification of the $\underline{\underline{A}}$ matrix.

Having performed this operation we have for the two regions:-

Region Ω^1 (Fluid)
$$\underline{F} = \underline{\underline{A}}\,\underline{U} \qquad \ldots\ldots\ldots(41)$$

The free surface unknowns have been eliminated by the reduction process.

All the unknowns are then on the interface Γ_I^1

Region Ω^2 (Structure)
$$\begin{bmatrix} \underline{F}_e + \underline{F}^2_I \\ \hline \underline{F}_e \end{bmatrix} = \begin{bmatrix} \underline{\underline{K}}^2_I & \underline{\underline{K}}^2 \end{bmatrix} \times \begin{bmatrix} \underline{U}^2_I \\ \underline{U}^2 \end{bmatrix} \ldots\ldots\ldots(42)$$

where $\underline{\underline{K}}^2_I$ and $\underline{\underline{K}}^2$ are the stiffness matrices for the structure corresponding to the interface and other nodes calculated using the finite element programs. In fact the master degrees of freedom are chosen so that the matrix equation (42) is reduced to involve only interfacial nodes. Hence we have:-

$$\underline{F}_e + \underline{F}^2_I = \underline{\underline{K}}^2_I \; \underline{U}^2_I \qquad \ldots\ldots\ldots(43)$$

Equations (41) and (43) may now be combined using the compatibility and equilibrium conditions (39) and (40) to give:-

$$\underline{F}_e = \left[\underline{\underline{K}}^2_I + \underline{\underline{A}}'\right] \underline{U}_I \qquad \ldots\ldots\ldots(44)$$

This equation may be solved to give the displacements in the usual way for given forcings $\underline{F}_e$. Because of the compressibility of the fluid and the different phases of the forcing functions the quantities $\underline{F}_e$ $\underline{U}_I$ and $\underline{\underline{A}}'$ are all complex.

RESULTS

The above analysis was performed for the pressure vessel, it was found that the acoustic resonance linked in with ovalling behaviour of the structure at the forcing frequency the real parts of the displacements are shown in Figures 7 and 8. The imaginary parts were obtained, and hence the full displacement time history was available. These displacements were then applied to part of the vessel and peak stresses were obtained. A fatigue analysis was then performed on the results.

The effect of the inclusion of the fluid in the modelling reduced the displacements in the structure by 10-15%. This figure could not however have been deduced without performance of the full analysis. The presence of the fluid has two conflicting effects.

1) It introduced a damping and added mass to the surface of the shell, reducing the expected deflections.

2) It enables energy to be transmitted across the shell, potentially increasing the interactions and expected deflections.

OTHER APPLICATIONS

By choosing our Green's function to represent the propagation properties of a disturbance of the fluid we can represent different kinds of fluid, the simplest being represented by the incompressible, potential Green's function.

Furthermore if we have a free surface we can choose our Green's function to satisfy the free surface boundary condition and in this way avoid the necessity of discretising the surface. In a similar way we may easily solve external problems such as wave diffraction problems with the boundary conditions at infinity, being satisfied by our Green's function. Alternatively free surface effects may be negligible and the normal potential functions may be used.

Flow problems may also be solved by this method so long as the boundary conditions on solid surfaces are suitably modified and the right choice of physical variables made.

Table 1 illustrates some situations which are amenable to solution by this method, (together with the governing equations and Green's functions associated with them).

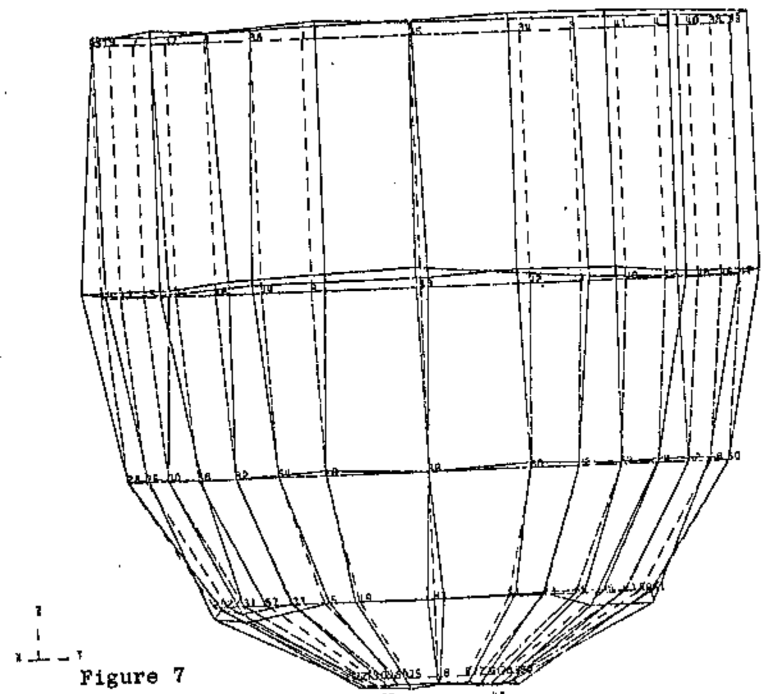

Figure 7

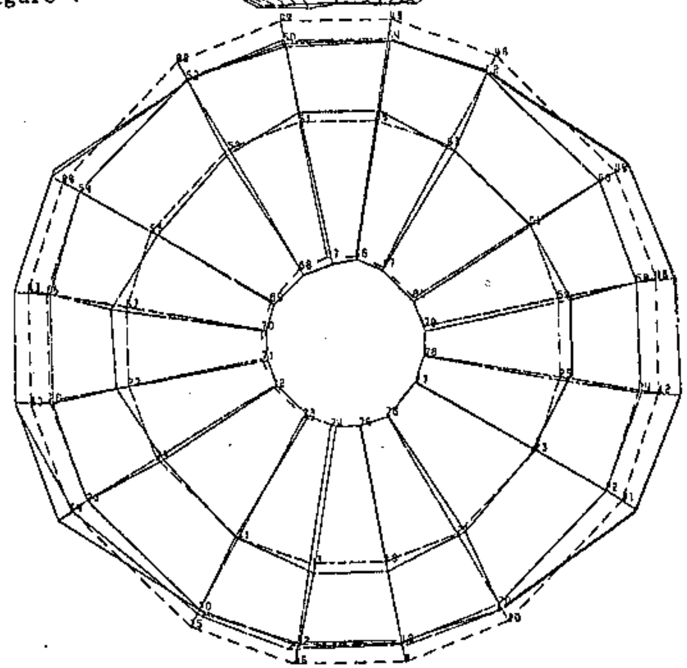

Figure 8 - Displacements of the operating structure

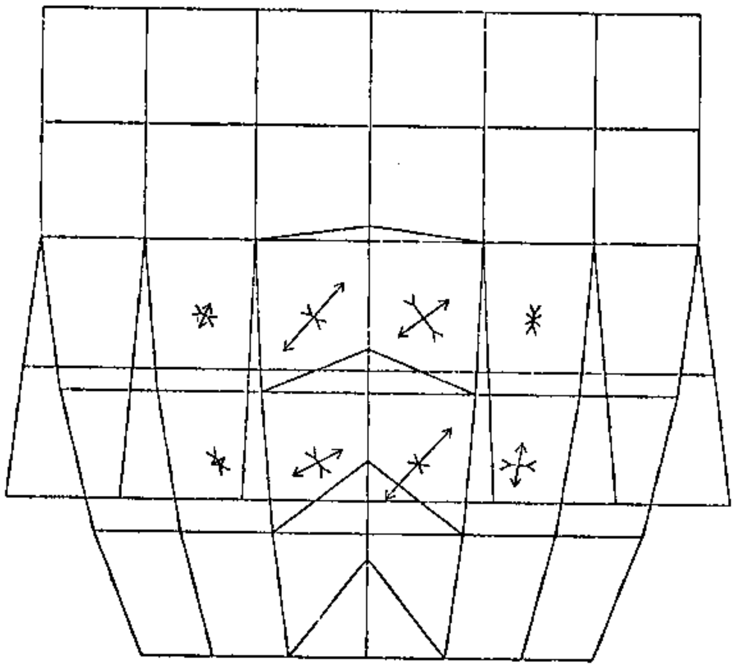

Figure 9 - Stresses in the Structure

The method is currently being applied to the problem of determining the change of response of offshore tethered platforms due to the presence of on-board storage tanks. This is an altogether simpler problem as the fluid is incompressible, the variables are real resulting in an added mass matrix due to the fluid. This matrix is frequency independent and can be used in the conventional normal mode/eigenfrequency calculations.

TABLE 1

CAPABILITIES OF THE PROGRAM

	With a Free Surface	Without a Free Surface
Compressible $G = \frac{e^{-i\kappa r}}{-4\pi r}(e^{i\omega t})$ $(\nabla^2 + \kappa^2) P = 0$	1. Storage Vessels 2. 2-phase flow in pipes 3. Earthquake problems (known displacements)	1. Acoustic noise from extended sources 2. Compressible flow in pipes 3. Air duct noise propagation 4. Panel flutter
Incompressible $G = -\frac{1}{4\pi r}$ $\nabla^2 P = 0$	1. Oil storage tanks, internal and external problem 2. Flow in pipes with free surface 3. Surface waves interior problem	1. Simplest cases of flow 2. External flow round pipes storage tanks etc. 3. Dynamics of risers

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SECTION IV

NUMERICAL TECHNIQUES